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A New Approach to Determine the Optimal Solution for Transportation Problems

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Abstract

One particular kind of linear programming problem is the transportation problem. It aims to reduce the cost of transporting goods from many supply points to different demand areas. Optimal solutions are typically found using techniques like the Stepping Stone method and the Modified Distribution (MODI) method. However, before optimality is examined, both of these approaches require the identification of an initial basic feasible solution. Conventional approaches can be time-consuming because they require iterations for both the initial basic feasible solution and the optimality evaluation. This study presents a novel approach to the transportation problem that does not require an initial basic feasible solution in order to identify the optimal solution. Both balanced and unbalanced transportation problems can be solved using this technique. The proposed method is appropriate for large transportation systems since it lowers computational complexity and provides a few iterations to optimality. Using numerical examples, we tested the proposed approach and compared the outcomes with the optimal solution obtained using the Vogel Approximation Method (VAM) and the MODI method. We applied MODI for the optimality check and VAM for the initial basic feasible solution. The outcomes were also compared to the optimal solution determined with the "TORA" software. Every strategy produces the same ideal result, confirming the precision and effectiveness of the suggested approach. This approach requires a straightforward mathematical and logical computation that is simpler to learn and apply.

Keywords: MODI method, TORA software, Transportation Problem.

Introduction

One of the initial and most significant uses of linear programming problems is the transportation problem. It focuses on the most economical method of meeting supply and demand restrictions while distributing a product from several suppliers (sources) to multiple consumers (destinations). It is assumed that there are balanced and unbalanced states in transportation problems. The primary objective of the problem is to determine an optimal schedule for shipping the commodity while satisfying demand at each destination. The modified Distribution (MODI) and Stepping Stone methods are the well-known traditional methods for finding the optimal solution to a transportation problem. (Soomro et al.,(2015)).

However, both of these methods to find optimal solutions require the determination of an initial basic feasible solution by using any of the available methods, such as “Northwest Corner”, “Least cost method”, and “Vogel’s approximation method” before the optimality is checked. This two-step process demands a higher number of iterations. So, there is a need for a more direct approach that bypasses the IBFS step without compromising accuracy. This research aims

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to provide a new approach that directly finds the optimal solution for transportation problems without requiring the initial basic feasible solution and with fewer iterations. Additional objectives are: to introduce a simple method, to minimize time, computational complexity, and effort in solving problems, to handle large transportation networks effectively, and to ensure the applicability to real-world transportation problems.

This research is significant as it offers a novel approach to solving transportation problems that reduces the computational load and the number of iterations required to reach the optimal solution. The proposed method's ability to deliver results more easily makes it particularly relevant in today's fast-paced, data-driven decision-making environments. Through this research, a more streamlined solution to transportation problems is introduced, with the potential to be adopted in real-world applications, offering both time and cost savings.

Formulation of the Problem:

Assume that there are m sources and n destinations. Let cij be the cost of moving one unit of commodity from source i to destination j. Let ai be the number of supply units offered at source i and bj be the number of demand units needed at destination j. If xij ($x_{ij} \geq 0$) is the number of units transported from source i to destination j. (Shaikh et al., (2018)).

$\sum_{j=1}^{n} x_{ij} \leq a_i$, (i=1,2,……,m), (Supply Constraints).
$\sum_{i=1}^{m} x_{ij} \geq b_j$, (j=1,2,……,n), (Demand Constraints).
$x_{ij} \geq 0$, (i=1,2,……..,m; j=1,2,… n)

Minimize $Z=\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_i$

Methodology

This method is applicable to both balanced and unbalanced transportation problems. Step

1: Construct the transportation table from the given transportation problem.

Step 2: Cross out the row minimum in each row. If there are multiple minimums in a row, cross out all of them.

Step 3: Check whether each column contains at least one of the crossed row minimums. If there are columns that do not contain any of the crossed row minimums, from the entries of such columns, subtract their corresponding crossed row minimum and cross out the column entry that has the minimum subtracted value. If some column entries have equal minimum subtracted values, cross out all of them.

Step 4: Select the first crossed entry (row-wise). Suppose $(i,j)^{th}$ crossed entry is selected, count the total number of crossed entries in the ith row and j^{th} column excluding $(i,j)^{th}$ crossed entry. Continue it for all crossed entries.

Step 5: Choose a crossed entry with the minimum count from Step 4 and allocate as much as possible amount to that cell. If a tie occurs, choose a $(k,l)^{th}$ crossed entry breaking the tie such that the total sum of all the elements in k^{th} row and l^{th} column is maximum and allocate as much as possible amount.

Step 6: After performing step 5, delete the satisfied row or column and adjust the supply or demand.

Step 7: Check whether the resultant table contains at least one crossed entry in each row and each column. If not, repeat step 2 to step 7, Otherwise repeat step 4 to step 7 until all the supplies and demands are satisfied.

Results and Discussion

Numerical Example (Shenoy, G.V., 1998. Linear Programming Methods and Applications. Page no:155-156, Problem 17) (UNBALANCED)

	D1	D2	D3	Supply
S1	20	10	40	200
S2	10	20	30	300
S3	10	10	20	100
Demand	100	100	200	

By using Vogel Approximation and Modified Distribution Method to solve:

- Finding Initial Basic Feasible Solution by Using Vogel Approximation

Method After 4 iterations, the Initial Basic Feasible Solution is

	D1	D2	D3	D4	Supply
S1	20	100 10	100 40	0	200
S2	100 10	20	30	200 0	300
S3	10	10	100 20	0	100
Demand	100	100	200	200	

TotalTransportation Cost=100*10+100*40+100*10+200*0+100*20=8000

- Optimality Check by using the Modified Distribution

  Method After 3 iterations, the Optimal solution is

	D1	D2	D3	D4	Supply
S1	20	100 10	40	100 0	200
S2	100 10	20	100 30	100 0	300
S3	10	10	100 20	0	100
Demand	100	100	200	200	

Total Transportation Cost= 100*10+100*0+100*10+100*30+100*0+100*20=7000

By using TORA Software to solve:

TRANSPORTATION MODEL OUTPUT SUMMARY

Title: Numerical Example 02
Final Iteration No.: 3
Objective Value (minimum cost) =7000.00

From	To	Amt Shipped	Obj Coeff	Obj Contrib
S1: S1	D2: D2	100	10.00	1000.00
S1: S1	D4: DummyD	100	0.00	0.00
S2: S2	D1: D1	100	10.00	1000.00
S2: S2	D3: D3	100	30.00	3000.00
S2: S2	D4: DummyD	100	0.00	0.00
S3: S3	D3: D3	100	20.00	2000.00

By using Proposed Method to solve:

	D1	D2	D3	D4	Supply
S1	20	100	10	40	0 200 100
S2	10		20	30	0 300
S3	10		10	20	0 100
Demand	100	1	0	200	200

Repeat from Step 4

	D1	D3	D4	Supply
S1	20	40	0	100
S2	10	30	0	300
S3	10	100 20	0	100
Demand	100	200 100	200	

Repeat from Step 2

	D1	D3	D4	Supply
S1	20	40	100 0	100
S2	100 10	100 30	100 0	300
Demand	100	100	200 100	

Total transportation cost= 100*10+100*20+100*0+100*10+100*30+100*0= 7000

The optimal solutions are calculated by using the Vogel and Modi method, TORA software and the proposed method. In the above results, we can observe all the solutions from different methods are same for problems but when solving by using proposed approach, the optimal solution is obtained directly without finding initial basic feasible solution in 3 iterations, while in the conventional method, firstly, there are 4 iterations for finding an initial basic feasible solution and 3 iterations for optimality check. The proposed approach has simple calculations and easier to get the optimal solution.

Also, we need only one algorithm that gives a direct optimal solution when using the proposed method.

Conclusion

In this study, we have presented an innovative approach for finding an optimal solution, and we conclude that this approach provides an optimal solution directly without finding the initial basic feasible solution. By eliminating the step of obtaining an initial basic feasible solution, the proposed method reduces the number of iterations required, making it more efficient and effective than the traditional methods. Also, we claim that the proposed approach reduces iterations even when the initial basic feasible solution is obtained by the most accurate method known as the Vogel approximation method and optimality check is done by the well-known Modi method. One of the attractive features of this proposed method is that it has only very simple arithmetical calculations like addition and subtraction, so that everyone can easily understand and use and that is very helpful for decision-makers who are involved with logistic and supply chain problems in the real world. Also, this approach reduces computational complexity, effort, time to solve problems and is suitable to handle large-scale problems easily. In addition, the ability of the new method to handle both balanced and unbalanced transportation problems without defects in accuracy demonstrates its practical potential in various industries.

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